Statistical Inference
Final Examination

2018/01/08

Time: 1:20 pm – 5:00 pm

1. (15%) Please prove

$$\text{(I)} = \sigma^2 + n^{-1}\boldsymbol{\beta}'\boldsymbol{X}'(\boldsymbol{I} - \boldsymbol{P}_\alpha)\boldsymbol{X}\boldsymbol{\beta} + o_p(1)$$

on P.6 of the note for CV.

2. (15%) Please prove

$$\text{(II)} = \frac{2d_\alpha\sigma^2}{n} + o_p(n^{-1})$$

on P.7 of the note for CV.

3. (20%) Please prove

$$\begin{aligned}(B) &\leq \left(\frac{n_v}{n_c}\right)^2 \frac{1}{n_v b} \max_{s\in\mathcal{B}_{n_v}} \|\hat{\boldsymbol{R}}_{\alpha,s} - \hat{\boldsymbol{R}}_{\alpha,s^c}\| \max_{s\in\mathcal{B}_{n_v}} \|\hat{\boldsymbol{R}}_{\alpha,s}^{-1}\| \sum_{s\in\mathcal{B}_{n_v}} \boldsymbol{\gamma}_{\alpha,s}'\boldsymbol{P}_{\alpha,s}\boldsymbol{\gamma}_{\alpha,s} \\ &= \left[\left(\frac{n_v}{n_c}\right)^2 \frac{1}{n_v b} \sum_{s\in\mathcal{B}_{n_v}} \boldsymbol{\gamma}_{\alpha,s}'\boldsymbol{P}_{\alpha,s}\boldsymbol{\gamma}_{\alpha,s}\right] o(1)\end{aligned}$$

on P.12 of the note for CV.

4. (20%) Please show

$$\left(\frac{n_v}{n_c}\right)^2 \frac{1}{n_v b} \sum_{s\in\mathcal{B}_{n_v}} \boldsymbol{\gamma}_{\alpha,s}'\boldsymbol{Q}_{\alpha,s}\boldsymbol{\gamma}_{\alpha,s} = \frac{d_\alpha\sigma^2}{n_c} + o_p(n_c^{-1})$$

on P.15 of the note for CV.

5. (20%) Please prove

$$E\left[\left(\frac{n_v}{n_c}\right)^2 \frac{1}{n_v b} \sum_{i=1}^{b} \boldsymbol{\gamma}_{\alpha,s_i}'\boldsymbol{Q}_{\alpha,s_i}\boldsymbol{\gamma}_{\alpha,s_i} \,\middle|\, (y_1,\boldsymbol{x}_1),\ldots,(y_n,\boldsymbol{x}_n)\right] = \frac{d_\alpha\sigma^2}{n_c} + o_p(n_c^{-1})$$

on P.4 of the note for MCCV.

6. (20%) Please prove

$$(\hat{\boldsymbol{\beta}}_\alpha - \boldsymbol{\beta}_\alpha)' \left(\frac{1}{n_v b} \sum_{i=1}^{b} \boldsymbol{X}_{\alpha,s_i}'\boldsymbol{X}_{\alpha,s_i}\right) (\hat{\boldsymbol{\beta}}_\alpha - \boldsymbol{\beta}_\alpha) = O_p(n^{-1})$$

on P.8 of the note for MCCV.

7. (25%) Please prove

$$\sum_{t=M+1}^{n} \boldsymbol{x}_{t,\alpha}'(\hat{\boldsymbol{\beta}}_{t-1,\alpha} - \boldsymbol{\beta}_\alpha)\epsilon_t(1 - d_{t,\alpha}) = o_p(\log n)$$

on P.5 of the note for APE.

8. (25%) Please prove

$$\sum_{t=M+1}^{n} (y_t - \boldsymbol{x}'_{t,\alpha}\hat{\boldsymbol{\beta}}_{t-1,\alpha})^2 \geq \boldsymbol{y}'(\boldsymbol{I} - \boldsymbol{P}_\alpha)\boldsymbol{y} + O_p(1)$$

on P.6 of the note for APE.